2 Polynomials

Fastrack Revision

▶ **Polynomial:** An expression of the form $p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n$, where $a_0 \neq 0$, is called a polynomial in the variable x of degree n.

Here, $a_0, a_1, a_2, ..., a_n$ are real numbers and each power of x (*i.e.*, n) is a non-negative integer.

For example, $13x + 5$, $19x^2 + 5x + 8$, $-23y^3 + 12y^2 + 8y + 6$, etc.

▶ **Degree of the Polynomial:** The highest power of x in a polynomial $p(x)$ is called the degree of the polynomial.

▶ **Types of Polynomials:** Main types of polynomials are given below:

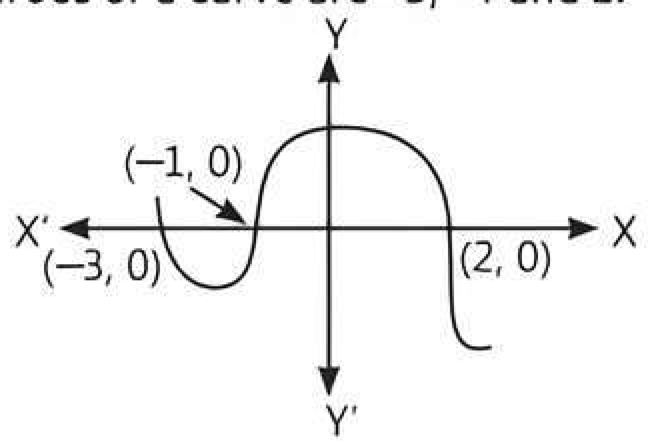
1. **Constant Polynomial:** A polynomial of degree zero (0) is called constant polynomial and it is of the form $p(x) = k$, where k is a constant.
   For example, $p(x) = 5$
2. **Zero Polynomial:** Constant polynomial 0 is called the zero polynomial.
   For example, $p(x) = 0x^3 - 0x + 0$
3. **Linear Polynomial:** A polynomial of degree 1 is called linear polynomial and it is of the form $p(x) = ax + b$, where a and b are real numbers and $a \neq 0$.
   For example, $p(x) = 5x + 3$
4. **Quadratic Polynomial:** A polynomial of degree 2 is called quadratic polynomial and it is of the form $p(x) = ax^2 + bx + c$, where a, b and c are real numbers and $a \neq 0$.
   For example, $p(x) = 3x^2 + 4x - 6$
5. **Cubic Polynomial:** A polynomial of degree 3 is called cubic polynomial and it is of the form $p(x) = ax^3 + bx^2 + cx + d$, where a, b, c and d are real numbers and $a \neq 0$.
   For example, $p(x) = 3x^3 + 5x^2 + 6x - 7$
6. **Biquadratic Polynomial:** A polynomial of degree 4 is called biquadratic polynomial and it is of the form $p(x) = ax^4 + bx^3 + cx^2 + dx + e$, where a, b, c, d and e are real numbers and $a \neq 0$.
   For example, $p(x) = 5x^4 - 3x^3 + 4x^2 + 8x + 15$

▶ **Value of a Polynomial at a Given Point:** If $p(x)$ is a polynomial in x and α is a real number, then the value obtained by putting $x = \alpha$ in $p(x)$ is called the value of $p(x)$ at $x = \alpha$.

▶ **Zero of a Polynomial:** A real number α is said to be a zero of a polynomial $p(x)$, if $p(\alpha) = 0$. Here, $(x - \alpha)$ is called the factor of the polynomial $p(x)$.

▶ **Geometrical Meaning of Zeroes of a Polynomial:** Whenever the curve intersect(s) the X-axis, the x-coordinate of that point(s) is/are the zeroes of the curve.

e.g. Here zeroes of a curve are -3, -1 and 2.



Knowledge BOOSTER

1. *A non-zero constant polynomial has no zero.*
2. *The degree of a zero polynomial is not defined.*
3. *A polynomial of degree 'n' can have atmost n zeroes. i.e., a quadratic polynomial can have atmost 2 zeroes and a cubic polynomial can have atmost 3 zeroes.*

▶ **Relationship between the Zeroes and the Coefficients of a Polynomial**

1. A quadratic polynomial whose zeroes are α and β, is given by
   $p(x) = k[x^2 - (\alpha + \beta)x + \alpha\beta] = k[x^2 - (\text{Sum of zeroes})\,x + (\text{Product of zeroes})]$,
   where k is the arbitrary constant.
2. Let α and β be the zeroes of a quadratic polynomial $p(x) = ax^2 + bx + c$, where $a \neq 0$, then
   $$\text{Sum of zeroes} = \alpha + \beta = (-1)\cdot\frac{b}{a} = (-1)\cdot\frac{(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$
   $$\text{Product of zeroes} = \alpha\beta = (-1)^2\cdot\frac{c}{a} = (-1)^2\cdot\frac{\text{Constant term}}{\text{Coefficient of } x^2}$$
3. A cubic polynomial whose zeroes are α, β and γ, is given by
   $p(x) = k[x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma]$
   $\Rightarrow p(x) = k[x^3 - (\text{Sum of zeroes})\,x^2 + (\text{Sum of the product of zeroes taken two at a time})\,x - (\text{Product of zeroes})]$,
   where k is the arbitrary constant.
4. If α, β and γ are the zeroes of a cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$, then
   $$\text{Sum of zeroes} = \alpha + \beta + \gamma = (-1)\cdot\frac{b}{a} = (-1)\cdot\frac{(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$$
   Sum of the product of zeroes taken two at a time
   $$= \alpha\beta + \beta\gamma + \gamma\alpha = (-1)^2\cdot\frac{c}{a} = (-1)^2\cdot\frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$
   $$\text{Product of zeroes} = \alpha\beta\gamma = (-1)^3\cdot\frac{d}{a} = (-1)^3\cdot\frac{(\text{Constant team})}{\text{Coefficient of } x^3}$$

Practice Exercise

Multiple Choice Questions

Q 1. The maximum number of zeroes a cubic polynomial can have, is: *[CBSE 2020]*

a. 1 b. 4 c. 2 d. 3

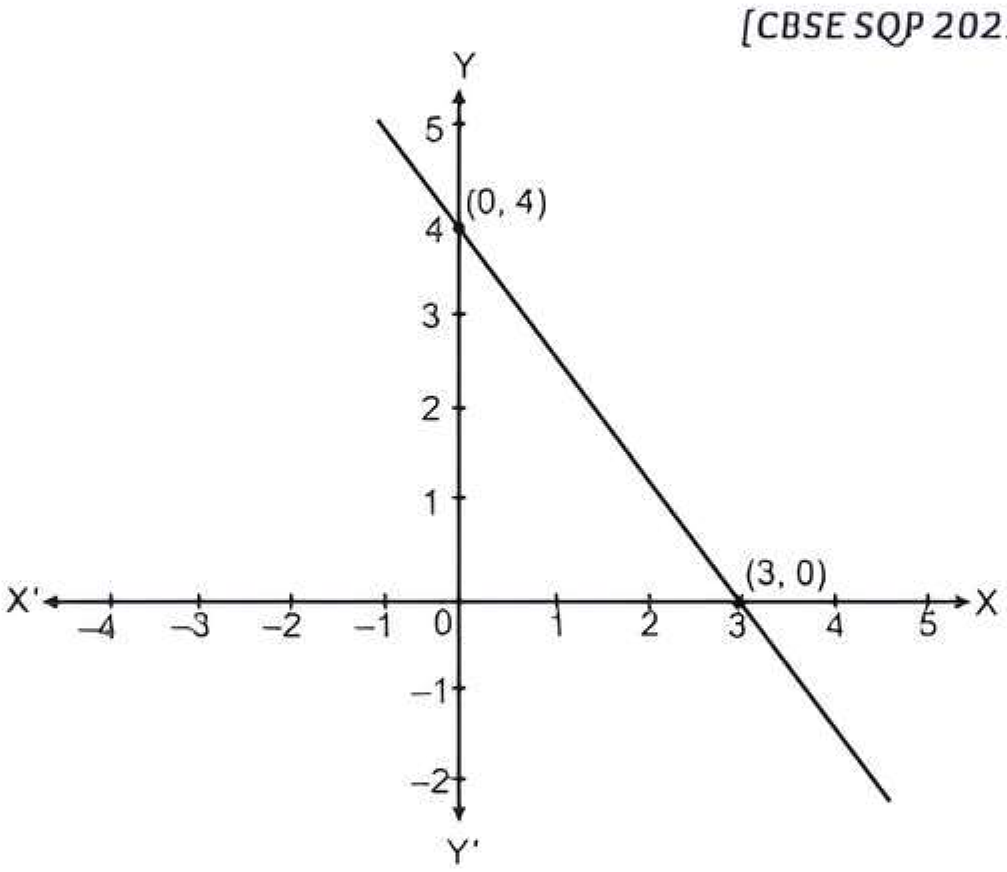
Q 2. The zeroes of the quadratic polynomial $x^2 + 25x + 156$ are:

a. both positive
b. both negative
c. one positive and one negative
d. Can't be determined

Q 3. The graph of a polynomial $p(x)$ cuts the X-axis at 3 points and touches it at 2 other points. The number of zeroes of $p(x)$ is: *[CBSE 2021 Term-I]*

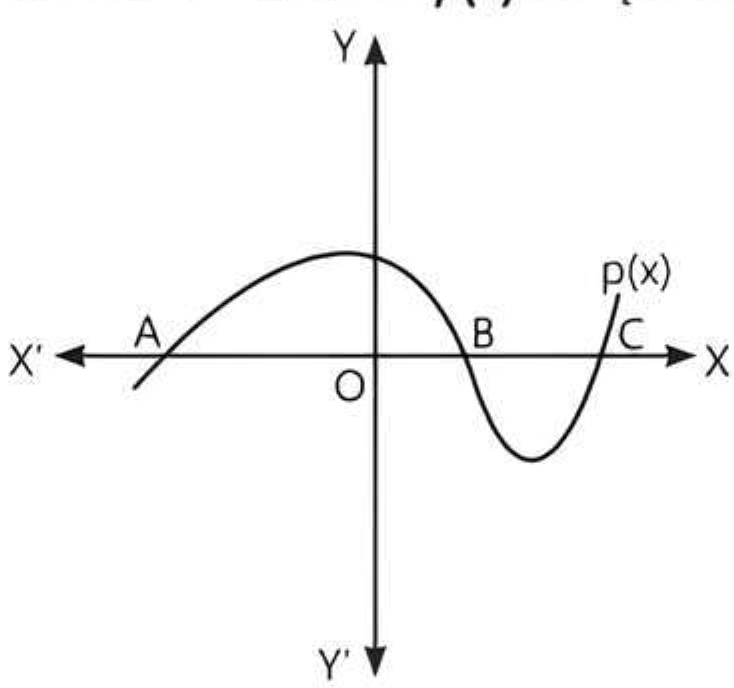
a. 1 b. 2 c. 3 d. 5

Q 4. The given linear polynomial $y = f(x)$ has: *[CBSE SQP 2023-24]*



a. 2 zeroes
b. 1 zero and the zero is '3'
c. 1 zero and the zero is '4'
d. no zero

Q 5. In figure, the graph of a polynomial $p(x)$ is shown. The number of zeroes of $p(x)$ is: *[CBSE 2021 Term-I]*



a. 1 b. 2 c. 3 d. 4

Q 6. If one zero of the polynomial $f(x) = 3x^2 + 11x + p$ is reciprocal of the other, then the value of p is:

a. 0 b. 3 c. $\frac{1}{3}$ d. -3

Q 7. If α, β are zeroes of the polynomial $x^2 - 1$, then value of $(\alpha + \beta)$ is: *[CBSE 2023]*

a. 2 b. 1
c. -1 d. 0

Q 8. A quadratic polynomial, the product and sum of whose zeroes are 5 and 8 respectively, is: *[CBSE 2021 Term-I]*

a. $k(x^2 - 8x + 5)$ b. $k(x^2 + 8x + 5)$
c. $k(x^2 - 5x + 8)$ d. $k(x^2 + 5x + 8)$

Q 9. If two zeroes of the polynomial $x^3 + x^2 - 9x - 9$ are 3 and -3, then its third zero is:

a. -1 b. 1 c. -9 d. 9

Q 10. If α, β are the zeroes of the polynomial $p(x) = 4x^2 - 3x - 7$, then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ is equal to: *[CBSE 2023]*

a. $\frac{7}{3}$ b. $\frac{-7}{3}$ c. $\frac{3}{7}$ d. $\frac{-3}{7}$

Q 11. If α, β are the zeroes of $f(x) = 2x^2 + 8x - 8$, then:

a. $\alpha + \beta = \alpha\beta$ b. $\alpha + \beta > \alpha\beta$
c. $\alpha + \beta < \alpha\beta$ d. $\alpha + \beta + \alpha\beta = 0$

Q 12. If the sum of the zeroes of the polynomial $p(x) = (p^2 - 23)x^2 - 2x - 12$ is 1, then p takes the value(s):

a. $\sqrt{23}$ b. -23 c. 2 d. ± 5

Q 13. If α, β are the zeroes of the quadratic polynomial $p(x) = x^2 - (k + 6)x + 2(2k - 1)$, then the value of k, if $\alpha + \beta = \frac{1}{2}\alpha\beta$, is: *[CBSE 2021 Term-I, 2019]*

a. -7 b. 7 c. -3 d. 3

Q 14. The number of quadratic polynomials having zeroes -5 and -3 is: *[CBSE 2023]*

a. 1 b. 2
c. 3 d. More than 3

Q 15. A quadratic polynomial, the sum of whose zeroes is 0 and one zero is 4, is:

a. $x^2 - 16$ b. $x^2 + 16$
c. $x^2 + 4$ d. $x^2 - 4$

Q 16. If α and β are the zeroes of the polynomial $ax^2 - 5x + c$ and $\alpha + \beta = \alpha\beta = 10$, then: *[CBSE 2023,16]*

a. $a = 5, c = \frac{1}{2}$ b. $a = 1, c = \frac{5}{2}$
c. $a = \frac{5}{2}, c = 1$ d. $a = \frac{1}{2}, c = 5$

Q 17. If α, β, γ are the zeroes of the polynomial $f(x) = x^3 - ax^2 + bx - c$, then $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} =$

a. $\frac{c}{a}$ b. $\frac{a}{c}$ c. $-\frac{a}{c}$ d. $-\frac{c}{a}$

Q 18. If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and −3, then: *[NCERT EXEMPLAR, CBSE 2023]*

a. $a = -7, b = -1$ b. $a = 5, b = -1$
c. $a = 2, b = -6$ d. $a = 0, b = -6$

Q 19. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it: *[NCERT EXEMPLAR]*

a. has no linear term and the constant term is negative
b. has no linear term and the constant term is positive
c. can have a linear term but the constant term is negative
d. can have a linear term but the constant term is positive

Q 20. The zeroes of the polynomial $x^2 - 3x - m(m+3)$ are: *[CBSE 2020]*

a. $m, m+3$ b. $-m, m+3$
c. $m, -(m+3)$ d. $-m, -(m+3)$

Q 21. If $x - 1$ is a factor of the polynomial $p(x) = x^3 + ax^2 + 2b$ and $a + b = 4$, then *[CBSE 2021 Term-I]*

a. $a = 5, b = -1$ b. $a = 9, b = -5$
c. $a = 7, b = -3$ d. $a = 3, b = 1$

Q 22. If a cubic polynomial with the sum of its zeroes, sum of the products two at a time and product of its zeroes as 0, −7 and −6 respectively, then the cubic polynomial is:

a. $x^3 + 7x - 6$ b. $x^3 + 7x + 6$
c. $x^3 - 7x - 6$ d. $x^3 - 7x + 6$

Q 23. If 2 and 1/2 are the zeroes of $px^2 + 5x + r$, then: *[CBSE 2021 Term-I]*

a. $p = r = 2$ b. $p = r = -2$
c. $p = 2, r = -2$ d. $p = -2, r = 2$

Q 24. If one zero of the polynomial $x^2 - 3kx + 4k$ be twice the other, then the value of k is: *[CBSE 2023]*

a. −2 b. 2 c. $\frac{1}{2}$ d. $-\frac{1}{2}$

Q 25. If α and β are the zeroes of the quadratic polynomial $p(x) = x^2 - ax - b$, then the value of $\alpha^2 + \beta^2$ is: *[CBSE 2023]*

a. $a^2 - 2b$ b. $a^2 + 2b$ c. $b^2 - 2a$ d. $b^2 + 2a$

Assertion & Reason Type Questions

Directions (Q. Nos. 26-30): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 26. Assertion (A): $f(x) = 2x^3 - \frac{3}{x} + 7$ is a polynomial in the variable x of degree 3.

Reason (R): The highest power of x in a polynomial $f(x)$ is called the degree of the polynomial $f(x)$.

Q 27. Assertion (A): The polynomial $p(x) = x^2 + 3x + 3$ has two real zeroes.

Reason (R): A quadratic polynomial can have at most two real zeroes. *[CBSE 2023]*

Q 28. Assertion (A): If one zero of the polynomial $p(x) = (k^2 + 9)x^2 + 9x + 6k$ is the reciprocal of the other zero, then $k = 3$.

Reason (R): If $(x - \alpha)$ is a factor of the polynomial $p(x)$, then α is a zero of $p(x)$.

Q 29. Assertion (A): If the sum and product of zeroes of a quadratic polynomial is 3 and −2 respectively, then the quadratic polynomial is $x^2 - 3x - 2$.

Reason (R): If S is the sum of zeroes and P is the product of zeroes of a quadratic polynomial, then the quadratic polynomial is given by $x^2 - Sx + P$.

Q 30. Assertion (A): If two zeroes of the polynomial $f(x) = x^3 - 2x^2 - 3x + 6$ are $\sqrt{3}$ and $-\sqrt{3}$, then its third zero is 4.

Reason (R): If α, β and γ are the zeroes of the polynomial $f(x) = ax^3 + bx^2 + cx + d$. Then,

$$\text{Sum of the zeroes} = -(1)\cdot\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

Fill in the Blanks Type Questions

Q 31. The polynomial of the form $ax^2 + bx + c, a = 0$ is of type ....

Q 32. If graph of a polynomial does not intersects the X-axis but intersects Y-axis in one point, then number of zeroes of the polynomial is equal to ....

Q 33. If $p(x) = ax^2 + bx + c$, then $-\frac{b}{a}$ is equal to .... of zeroes.

Q 34. Suppose two zeroes of cubic polynomial $ax^3 + bx^2 + cx + d$ are zero. The third zero is .... *[NCERT EXERCISE]*

True/False Type Questions

Q 35. A biquadratic polynomial have atmost four zeroes.

Q 36. If one of the factor of $x^2 + x - 20$ is $(x + 5)$, then the other factor is $(x + 4)$.

Q 37. If $(x + 2)$ is a factor of $x^3 - 2ax^2 + 16$, then the value of a is 4.

Q 38. Graph of a quadratic polynomial is an ellipse.

Q 39. The quadratic polynomial whose sum of zeroes is 5 and product of zeroes is -2, is $x^2 - 5x - 2$.

Solutions

1. (d) The maximum number of zeroes of a cubic polynomial can have 3.
2. (b) Let α and β be the zeroes of $x^2 + 25x + 156$.

   Then, $\alpha + \beta = -25$ and $\alpha\beta = 156$

   This happens when α and β are both negative.
3. (d)

TiP

A curve either touches or intersects the X-axis, then it has zeroes of polynomial.

The number of zeroes of $p(x)$

= Number of intersection points + Number of touches points

$= 3 + 2 = 5$

4. (b) The number of zeroes of the polynomial $f(x)$ is one as the graph intersects the X-axis at one point only. So. $f(x)$ has 1 zero and the zero is 3.
5. (c) Given, curve intersect the X-axis at three points, then number of zeroes of $p(x)$ is 3.
6. (b) Let α and $\frac{1}{\alpha}$ be the zeroes of $f(x) = 3x^2 + 11x + p$.

   $\therefore$ Product of zeroes $= \alpha \cdot \frac{1}{\alpha} = \frac{p}{3}$

   $\Rightarrow \quad p = 3$
7. (d) Given, α and β are the zeroes of a polynomial $f(x) = x^2 - 1 = x^2 + 0 \cdot x - 1$.

   Then, $\alpha + \beta = (-1) \cdot \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

   $= -\frac{0}{1} = 0$
8. (c) Given, sum of zeroes = 8

   and product of zeroes = 5

TR!CK

Quadratic polynomial = k[x²–(sum of zeroes)x + (product of zeroes)]

$\therefore$ Required quadratic polynomial

$= k\,[x^2 - (\text{sum of zeroes})x + (\text{product of zeroes})]$

$= k\,[x^2 - 5x + 8]$

9. (a) Let the third zero of the polynomial $x^3 + x^2 - 9x - 9$ be α.

   Now, sum of zeroes $= (-1) \cdot \frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

   $= \frac{-1}{1} = -1$

   $\Rightarrow \quad 3 + (-3) + \alpha = -1 \Rightarrow \alpha = -1$
10. (d) Given, α and β are the zeroes of the polynomial $p(x) = 4x^2 - 3x - 7$.

    $\therefore \quad \alpha + \beta = (-1) \cdot \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{(-3)}{4} = \frac{3}{4}$

    and $\alpha\beta = (-1)^2 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^2} = 1 \cdot \frac{(-7)}{4} = -\frac{7}{4}$

    So, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{3}{4} \times \left(\frac{-4}{7}\right) = -\frac{3}{7}$
11. (a) Since, α and β are the zeroes of $2x^2 + 8x - 8$.

    $\therefore \quad \alpha + \beta = -\frac{8}{2} = -4$ and $\alpha\beta = -\frac{8}{2} = -4$

    Hence, $\alpha + \beta = \alpha\beta$
12. (d) Let α and β be the zeroes of the polynomial $p(x) = (p^2 - 23)x^2 - 2x - 12$

    Then, $\alpha + \beta = -\frac{(-2)}{p^2 - 23} = \frac{2}{p^2 - 23}$

    Also, sum of zeroes $= \alpha + \beta = 1$ (given)

    $\Rightarrow \quad p^2 - 23 = 2$

    $\Rightarrow \quad p^2 = 25$

    $\Rightarrow \quad p = \pm 5$
13. (b) Given, quadratic polynomial is $p(x) = x^2 - (k + 6)x + 2(2k - 1)$

    Now, sum of zeroes, $\alpha + \beta = (-1) \cdot \frac{\text{Constant term of } x}{\text{Coefficient of } x^2}$

    $= -\frac{-(k + 6)}{1}$

    $= k + 6$

    and product of zeroes, $\alpha\beta = (-1) \cdot \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

    $= \frac{2(2k - 1)}{1}$

    $= 2(2k - 1)$

    Also given, $\alpha + \beta = \frac{1}{2}\alpha\beta$

    $\therefore \quad k + 6 = \frac{1}{2}[2(2k - 1)]$

    $\Rightarrow \quad k + 6 = (2k - 1)$

    $k = 7$
14. (d) Let $p(x) = ax^2 + bx + c$ be the required polynomial whose zeroes are -5 and -3.

    $\therefore$ Sum of zeroes $= -\frac{b}{a} = -5 + (-3) = \frac{-8}{1}$ ...(1)

and product of zeroes $= \frac{c}{a} = (-5) \times (-3) = \frac{15}{1}$ ...(2)

From eqs. (1) and (2),

$a = 1, b = -8$ and $c = 15$

$\therefore \quad p(x) = ax^2 + bx + c = x^2 - 8x + 15$

But we know that, if we multiply/divide any polynomial by any arbitrary constant, then the zeroes of polynomial never change.

$\therefore \quad p(x) = k[x^2 - 8x + 15]$(where k is a real number)

$\Rightarrow p(x) = \frac{1}{k}(x^2 - 8x + 15)$

(where, k is a non-zero real number)

Hence, the required number of polynomials are infinite *i.e.*, more than 3.

15. (a) Let α and β be the zeroes of the quadratic polynomial.

Then, $\alpha + \beta = 0$ (given)

$\Rightarrow \quad \alpha + 4 = 0$ [$\because$ one zero is 4 (given)]

$\Rightarrow \quad \alpha = -4$

$\therefore$ Zeroes of the polynomial are 4 and -4.

Now, sum of the zeroes $= 4 + (-4) = 0$

and product of the zeroes $= 4 \times (-4) = -16$

$\therefore$ The required polynomial

$= x^2 -$ (Sum of the zeroes)$x +$ Product of the zeroes

$= x^2 - 0x + (-16)$

$= x^2 - 16$

16. (d) Given, α and β are the zeroes of the polynomial $f(x) = ax^2 - 5x + c$.

$\therefore \alpha + \beta = (-1) \cdot \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = (-1) \cdot \frac{(-5)}{a} = \frac{5}{a}$ ...(1)

and $\alpha\beta = (-1)^2 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^2} = 1 \cdot \frac{c}{a} = \frac{c}{a}$ ...(2)

Also, given condition,

$\alpha + \beta = \alpha \cdot \beta = 10$

$\Rightarrow \quad \frac{5}{a} = \frac{c}{a} = 10$ [from eqs. (1) and (2)]

Taking, $\frac{5}{a} = 10 \Rightarrow a = \frac{5}{10} = \frac{1}{2}$.

Again taking, $\frac{c}{a} = 10 \Rightarrow \frac{c}{1/2} = 10 \Rightarrow c = 10 \times \frac{1}{2} = 5$

17. (b) Since, α, β and γ are the zeroes of the polynomial

$f(x) = x^3 - ax^2 + bx - c$.

$\therefore \quad \alpha + \beta + \gamma = -\frac{(-a)}{1} = a, \alpha\beta\gamma = -\frac{(-c)}{1} = c$

Now, $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = \frac{\alpha + \beta + \gamma}{\alpha\beta\gamma} = \frac{a}{c}$

18. (d) Let $p(x) = x^2 + (a + 1)x + b$

Given that, 2 and -3 are the zeroes of the quadratic polynomial $p(x)$.

$\therefore \quad p(2) = 0$ and $p(-3) = 0$

Now, $p(2) = 0$

$\Rightarrow \quad 2^2 + (a + 1)(2) + b = 0$

$\Rightarrow \quad 2a + b = -6$ ...(1)

Also, $p(-3) = 0$

$\Rightarrow \quad (-3)^2 + (a + 1)(-3) + b = 0$

$\Rightarrow \quad 3a - b = 6$ ...(2)

On solving eqs. (1) and (2), we get

$a = 0, b = -6$

19. (a) Let $p(x) = x^2 + ax + b$

and by given condition, the zeroes are α and $-\alpha$.

$\therefore$ Sum of the zeroes $= \alpha - \alpha = 0$

$\Rightarrow \quad a = 0 \Rightarrow p(x) = x^2 + b$, which cannot be linear

and product of zeroes, $\alpha(-\alpha) = b \Rightarrow -\alpha^2 = b$

which is only possible when $b < 0$.

Hence, it has no linear term and the constant term is negative.

20. (b) Let $f(x) = x^2 - 3x - m(m + 3)$

$= x^2 - \{(m + 3) - m\}x - m(m + 3)$

$= x^2 - (m + 3)x + mx - m(m + 3)$

$= x\{x - (m + 3)\} + m\{x - (m + 3)\}$

$= \{x - (m + 3)\}(x + m)$

For zeroes of $f(x)$, put

$\{x - (m + 3)\}(x + m) = 0$

$\Rightarrow \quad x - (m + 3) = 0$ or $x + m = 0$

$\Rightarrow \quad x = -m, (m + 3)$

21. (b) Given, $p(x) = x^3 + ax^2 + 2b$

Since, $x - 1$ is a factor of $p(x)$.

Therefore, $p(1) = 0$

$\Rightarrow \quad (1)^3 + a(1)^2 + 2b = 0$

$\Rightarrow \quad a + 2b = -1$

Also given, $a + b = 4$

On solving, we get

$a = 9$ and $b = -5$

22. (d) Let α, β and γ be the zeroes of the required polynomial.

Then, $\alpha + \beta + \gamma = 0$

$\alpha\beta + \beta\gamma + \alpha\gamma = -7$

$\alpha\beta\gamma = -6$

$\therefore$ Required cubic polynomial

$= k[x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma]$

(where k is non-zero constant)

$= k[x^3 + (0)x^2 + (-7)x - (-6)] = x^3 - 7x + 6$

(consider, $k = 1$)

23. (b) Let polynomial $f(x) = px^2 + 5x + r$

Given, 2 and $\frac{1}{2}$ are the zeroes of $f(x)$.

Now, sum of zeroes $= (-1) \cdot \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$\Rightarrow \quad 2 + \frac{1}{2} = \frac{-5}{p}$

$\Rightarrow \quad \frac{5}{2} = -\frac{5}{p} \quad \Rightarrow p = -2$

And product of zeroes $= (-1)^2 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\Rightarrow \quad 2\times\frac{1}{2}=\frac{r}{p}$

$\Rightarrow \quad 1=\frac{r}{-2} \quad \Rightarrow r=-2$

$\therefore \quad p=r=-2$

24. (b) Let α and β be the zeroes of the quadratic polynomial $p(x)=x^2-3kx+4k$.

According to question,

$$\alpha=2m \text{ and } \beta=m$$

$$\alpha+\beta=(-1)\cdot\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$\Rightarrow \quad 2m+m=-1\times\frac{(-3k)}{1}$

$\Rightarrow \quad 3m=3k \Rightarrow m=k$ ...(1)

and $\quad \alpha\beta=(-1)^2\cdot\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\Rightarrow \quad 2m\cdot m=1\times\frac{4k}{1} \Rightarrow 2m^2=4k$

$\Rightarrow \quad 2k^2=4k$ [from eq. (1)]

$\Rightarrow \quad 2k^2-4k=0 \quad \Rightarrow \quad 2k(k-2)=0$

$\Rightarrow \quad k=2, 0$

But $\quad k\neq 0.$

$\therefore \quad k=2$

25. (b) Given, α and β are the zeroes of the quadratic polynomial $p(x)=x^2-ax-b$.

$\therefore \quad \alpha+\beta=(-1)\cdot\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}=(-1)\cdot\frac{(-a)}{1}=a$

and $\quad \alpha\beta=(-1)^2\cdot\frac{\text{Constant term}}{\text{Coefficient of } x^2}=1\cdot\frac{(-b)}{1}=-b$

So, $\alpha^2+\beta^2=(\alpha+\beta)^2-2(\alpha\beta)=a^2-2(-b)=a^2+2b$

26. (d) **Assertion (A):** $f(x)=2x^3-\frac{3}{x}+7=2x^3-3x^{-1}+7$

is not a polynomial as one of the term is $-3x^{-1}$ in which the power of x is negative.

So, Assertion (A) is false.

Reason (R): It is true to say that the highest power of x in a polynomial $f(x)$ is the degree of the polynomial $f(x)$.

Hence, Assertion (A) is false but Reason (R) is true.

27. (d) **Assertion (A):** We have,

$$p(x)=x^2+3x+3$$

For finding zeroes, put

$$p(x)=0 \Rightarrow x^2+3x+3=0$$

Compare with $ax^2+bx+c=0$, we get

$$a=1, b=3, c=3$$

$\therefore$ Discriminant $(D)=b^2-4ac$

$$=(3)^2-4\times1\times3=9-12=-3<0$$

$\Rightarrow p(x)$ has no real zero.

So, Assertion (A) is false.

Reason (R): It is true to say that a quadratic polynomial has atmost 2 zeroes.

Hence, Assertion (A) is false but Reason (R) is true.

28. (b) **Assertion (A):** Let α and $\frac{1}{\alpha}$ be the zeroes of polynomial $p(x)=(k^2+9)x^2+9x+6k$.

Then, product of zeroes $=\alpha\times\frac{1}{\alpha}=\frac{6k}{k^2+9}$

$\Rightarrow \quad \frac{6k}{k^2+9}=1$

$\Rightarrow \quad k^2+9=6k \Rightarrow k^2-6k+9=0$

$\Rightarrow \quad (k-3)^2=0 \quad \Rightarrow k-3=0 \Rightarrow k=3$

Therefore, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

29. (a) **Assertion (A):** Let α and β be the zeroes of quadratic polynomial.

Now, given $\alpha+\beta=3=S$ and $\alpha\beta=-2=P$

$\Rightarrow \quad S=3$ and $P=-2$

$\therefore$ The required quadratic polynomial

$$=x^2-(S)x+P=x^2-3x-2$$

Therefore, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

30. (d) **Assertion (A):** Let $\alpha=\sqrt{3}, \beta=-\sqrt{3}$ be the zeroes of the polynomial $f(x)=x^3-2x^2-3x+6$ and γ be its third zero.

Then, $\quad \alpha+\beta+\gamma=-\left(\frac{-2}{1}\right)$

$\Rightarrow \quad \sqrt{3}-\sqrt{3}+\gamma=2 \quad \Rightarrow \gamma=2$

Therefore, Assertion (A) is false.

Reason (R): It is also true.

Hence, Assertion (A) is false but Reason (R) is true.

31. In polynomial ax^2+bx+c, if $a=0$, then given polynomial becomes $bx+c$, which is linear type.

32. Zero

33. In polynomial $p(x)=ax^2+bx+c$

$\therefore$ Sum of zeroes $=-\frac{b}{a}$

34. Let three zeroes of given polynomial $p(x)$ be α, β and γ. Then consider $\alpha=0$ and $\beta=0$

Now, sum of zeroes, $\alpha+\beta+\gamma=-\frac{b}{a}$

$\Rightarrow \quad 0+0+\gamma=-\frac{b}{a}$

$\Rightarrow \quad \gamma=-\frac{b}{a}$

Hence, third zero is $-\frac{b}{a}$.

35. True

36. Let $p(x) = x^2 + x - 20$

$= x^2 + 5x - 4x - 20$

$= x(x+5) - 4(x+5)$

$= (x-4)(x+5)$

Hence, given statement is false.

37. Let $p(x) = x^3 - 2ax^2 + 16$

Since, $(x+2)$ is a factor of $p(x)$. Therefore $p(-2) = 0$

$\therefore \quad (-2)^3 - 2a(-2)^2 + 16 = 0$

$\Rightarrow \quad -8 - 8a + 16 = 0$

$\Rightarrow \quad 8 - 8a = 0$

$\Rightarrow \quad a = 1$

Hence, given statement is false.

38. Graph of a quadratic polynomial is parabola.
Hence, given statement is false.

39. Given, sum of zeroes = 5
and product of zeroes = −2

$\therefore$ Quadratic polynomial

$= x^2 -$ (Sum of zeroes)x + (Product of zeroes)

$= x^2 - 5x + (-2) = x^2 - 5x - 2$

Hence, given statement is true.

Case Study Based Questions

Case Study 1

The below pictures show few natural examples of parabolic shape which can be represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola. In structures, there curves helps in loading and delivering that is why these curves are found in bridges and in architecture.

Based on the above information, solve the following questions:

Q 1. In the standard form of quadratic polynomial $ax^2 + bx + c$, a, b and c are:

a. all are real numbers

b. all are rational numbers

c. 'a' is a non-zero real number while b and c are any real numbers

d. all are integers

Q 2. If α and $\frac{1}{\alpha}$ are the zeroes of the quadratic polynomial $2x^2 - x + 8k$, then k is:

a. 4 b. $\frac{1}{4}$ c. $\frac{-1}{4}$ d. 2

Q 3. The graph of $x^2 + 4 = 0$:

a. intersects X-axis at two distinct points

b. touches X-axis at a point

c. neither touches nor intersects X-axis

d. either touches or intersects X-axis

Q 4. If the sum of the zeroes is $-p$ and product of the zeroes is $-\frac{1}{p}$, then the quadratic polynomial is:

a. $k\left(-px^2 + \frac{x}{p} + 1\right)$ b. $k\left(px^2 - \frac{x}{p} - 1\right)$

c. $k\left(x^2 + px - \frac{1}{p}\right)$ d. $k\left(x^2 - px + \frac{1}{p}\right)$

Q 5. The maximum number of zeroes of cubic polynomial has:

a. 0 b. 1 c. 2 d. 3

Solutions

1. A polynomial in the form of $ax^2 + bx + c$, where a, b and c are real numbers and $a \neq 0$, is known as a quadratic polynomial.
So, option (c) is correct.

2. Given that, α and $\frac{1}{\alpha}$ are the zeroes of the quadratic polynomial $2x^2 - x + 8k$.

$\therefore$ Product of zeroes $= (-1)^2 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

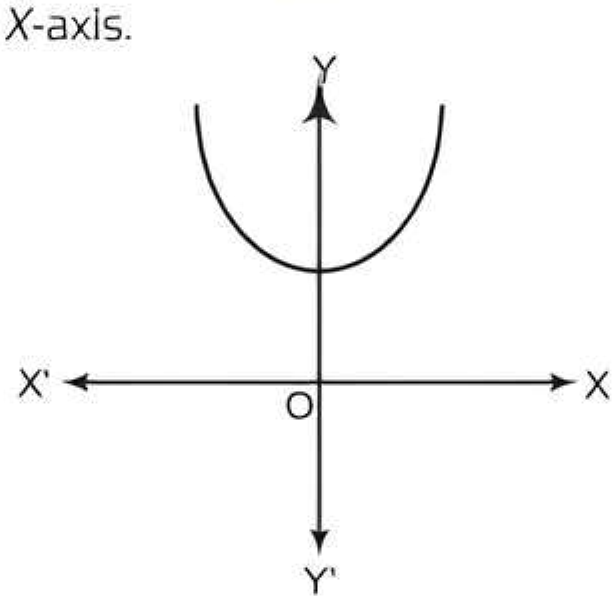
$\Rightarrow \quad \alpha \times \frac{1}{\alpha} = \frac{8k}{2} \quad \Rightarrow \quad 8k = 2 \Rightarrow k = \frac{1}{4}$

So, option (b) is correct.

3. We have $x^2 + 4 = 0$

Here, x^2 is positive, so the general shape of graph is 'U'. Since, the coefficient of x is zero, so the graph is central about the Y-axis. The constant part of the equation is of value +4, so it lifts the vertex up from $y = 0$ to $y = 4$.

$\therefore$ The graph of $x^2 + 4 = 0$, neither touches nor intersects X-axis.



So, option (c) is correct.

4. Given that, sum of the zeroes $= -p$

and product of the zeroes $= -\frac{1}{p}$

∴ Required quadratic polynomial

$= k\,[x^2 - (\text{Sum of the zeroes})\,x + \text{Product of the zeroes}]$

$= k\left[x^2 - (-p)x + \left(-\frac{1}{p}\right)\right] = k\left[x^2 + px - \frac{1}{p}\right]$

where, k is an arbitrary constant.

So, option (c) is correct.

5. The maximum number of zeroes of cubic polynomial has 3.

So, option (d) is correct.

Case Study 2

Asana is a body posture, originally and still a general term for a sitting meditation pose and later extended in hatha yoga and modern yoga as exercise, to any type of pose or position, adding reclining, standing, inverted, twisting and balancing poses. In the figure, one can observe that poses can be related to representation of quadratic polynomial.


Based on the above information, solve the following questions:

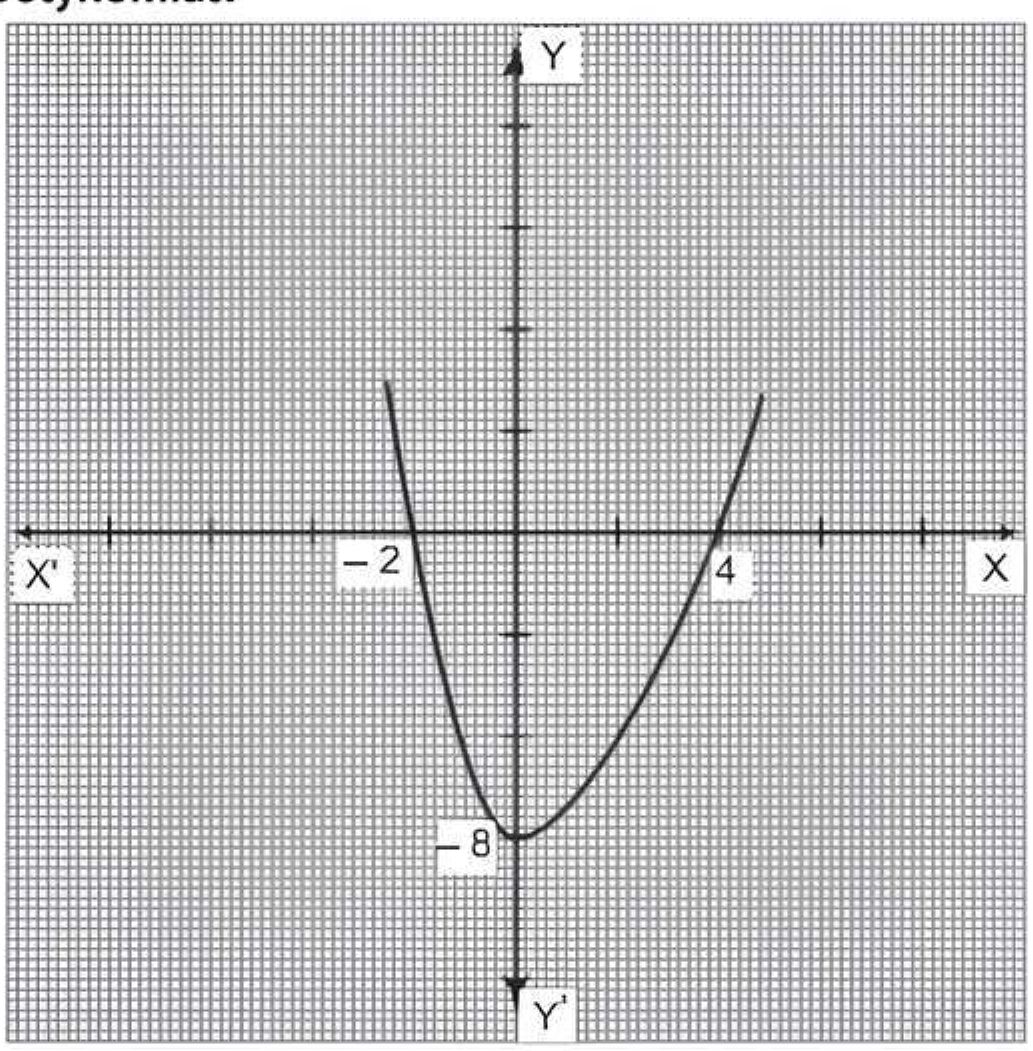
Q 1. The shape of the poses shown is:

a. spiral b. ellipse c. linear d. parabola

Q 2. The graph of parabola opens downward, if ....

a. $a \geq 0$ b. $a = 0$ c. $a < 0$ d. $a > 0$

Q 3. In the graph, how many zeroes are there for the polynomial?



a. 0 b. 1 c. 2 d. 3

Q 4. The quadratic polynomial of the two zeroes in the above shown graph are:

a. $k(x^2 - 2x - 8)$ b. $k(x^2 + 2x - 8)$

c. $k(x^2 + 2x + 8)$ d. $k(x^2 - 2x + 8)$

Q 5. The zeroes of the quadratic polynomial $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ are:

a. $\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$ b. $-\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

c. $\frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{4}$ d. $-\frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{4}$

Solutions

1. The shape of the poses shown is parabola which is a plane curve, mirror symmetrical and approximately ∪ or ∩ shaped.

So, option (d) is correct.

2. If $a < 0$ in $f(x) = ax^2 + bx + c$, the parabola opens downward.

So, option (c) is correct.

3.

TR!CK

The zeroes of the polynomial are the x-coordinates of those points where its graph touches or intersects the X-axis.

Here, the graph cut X-axis at two distinct points –2 and 4. Therefore, two zeroes are there for the polynomial.

So, option (c) is correct.

4. In the given graph, the graph cuts X-axis at points –2 and 4, which are the zeroes of the graph.

Now, the required polynomial

$= k\,[x^2 - (\text{Sum of zeroes})\,x + \text{Product of zeroes}]$

$= k\,[x^2 - (-2 + 4)\,x + (-2 \times 4)]$

$= k\,(x^2 - 2x - 8)$, where k is an arbitrary constant.

So, option (a) is correct.

5. Given, polynomial $= 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

$= 4\sqrt{3}x^2 + (8 - 3)x - 2\sqrt{3}$

(by splitting the middle term)

$= 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$

$= 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$

$= (\sqrt{3}x + 2)\,(4x - \sqrt{3})$

For the zeroes of the polynomial,

$(\sqrt{3}x + 2)\,(4x - \sqrt{3}) = 0$

$\Rightarrow \sqrt{3}x + 2 = 0$ or $4x - \sqrt{3} = 0$

$\Rightarrow x = \frac{-2}{\sqrt{3}}$ or $x = \frac{\sqrt{3}}{4} \Rightarrow x = \frac{-2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

So, option (b) is correct.

Case Study 3

The figure given shows the path of a diver, when she takes a jump from the diving board. Clearly it is a parabola.

Annie was standing on a diving board, 48 feet above the water level. She took a dive into the pool. Her height (in feet) above the water level at any time, 't' in seconds is given by the polynomial $h(t)$ such that $h(t) = -16t^2 + 8t + k$.

Based on the above information, solve the following questions:

Q 1. What is the value of k?

a. 0 b. -48 c. 48 d. $48/-16$

Q 2. At what time will she touch the water in the pool?

a. 30 sec b. 2 sec

c. 1.5 sec d. 0.5 sec

Q 3. Rita's height (in feet) above the water level is given by another polynomial $p(t)$ with zeroes -1 and 2. Then $p(t)$ is given by:

a. $t^2 + t - 2$ b. $t^2 + 2t - 1$

c. $24t^2 - 24t + 48$ d. $-24t^2 + 24t + 48$

Q 4. A polynomial $q(t)$ with sum of zeroes as 1 and the product as -6 is of Anu's height (in feet) above the water at any time t (in seconds). Then, $q(t)$ is given by:

a. $t^2 + t + 6$ b. $t^2 + t - 6$

c. $-8t^2 + 8t + 48$ d. $8t^2 - 8t + 48$

Q 5. The zeroes of the polynomial

$$r(t) = -12t^2 + (k-3)t + 48$$

are negative of each other. Then k is:

a. 3 b. 0 c. -1.5 d. -3

Solutions

1. Given, Annie was standing on a diving board, 48 feet above the water level.

Annie's height (in feet) above the water level at any time 't' (in seconds) is given by the polynomial $h(t)$ such that

$$h(t) = -16t^2 + 8t + k \quad ...(1)$$

Initially, at $t = 0$, Annie's height is 48 feet.

$\therefore \quad h(0) = 48$ feet

$\Rightarrow \quad -16(0)^2 + 8 \times 0 + k = 48 \Rightarrow k = 48$

Hence, option (c) is correct.

2. When Annie touches the pool, her height is 0 feet *i.e.*, $-16t^2 + 8t + 48 = 0$ above water level. $(\because k = 48)$

$\therefore \quad 2t^2 - t - 6 = 0$

$\Rightarrow \quad 2t^2 - 4t + 3t - 6 = 0$

$\Rightarrow \quad 2t(t-2) + 3(t-2) = 0$

$\Rightarrow \quad (2t+3)(t-2) = 0$

i.e. $\quad t = 2$ or $t = -3/2$

Since, time cannot be negative, so $t = 2$ sec.

Hence, option (b) is correct.

3. Given, $t = -1$ and $t = 2$ are the two zeroes of the polynomial $p(t)$.

Then $p(t) = k\{t - (-1)\}\{t - 2\}$

Rita's height, $p(t) = k(t+1)(t-2)$

Initially, at $t = 0$, Rita's height is 48 feet.

$\therefore \quad p(0) = 48$ feet

$\Rightarrow \quad 48 = k(0+1)(0-2)$

$\Rightarrow \quad -2k = 48 \Rightarrow k = -24$

$\therefore$ Required polynomial $p(t) = -24(t+1)(t-2)$

$= -24(t^2 - t - 2)$

$= -24t^2 + 24t + 48$

Hence, option (d) is correct.

4. Given, a polynomial $q(t)$ with sum of zeroes as 1 and product of zeroes as -6 is

$q(t) = k[t^2 - (\text{Sum of zeroes})\, t + (\text{Product of zeroes})]$

$= k[t^2 - (1)\, t + (-6)] = k(t^2 - t - 6)$

$\therefore$ Anu's height $q(t) = k(t^2 - t - 6)$

Initially, at $t = 0$, Anu's height is 48 feet.

$\therefore \quad q(0) = 48$ feet

$\Rightarrow \quad k(0 - 0 - 6) = 48$

$\Rightarrow \quad k = \dfrac{-48}{6} = -8$

$\therefore$ Required polynomial $q(t) = -8(t^2 - t - 6)$

$= -8t^2 + 8t + 48$

Hence, option (c) is correct.

5. Given polynomial, $r(t) = -12t^2 + (k-3)t + 48$

The zeroes of the above polynomial $r(t)$ are negative of each other *i.e.*, (α and $-\alpha$).

$\therefore$ Sum of zeroes $= -(1) \cdot \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$\Rightarrow \quad -\alpha + \alpha = -\dfrac{(k-3)}{(-12)}$

$\Rightarrow \quad 0 = \dfrac{k-3}{12} \Rightarrow k - 3 = 0$

$\Rightarrow \quad k = 3$

Hence, option (a) is correct.

Case Study 4

Ramesh was asked by one of his friends Anirudh to find the polynomial whose zeroes are $\dfrac{-2}{\sqrt{3}}$ and $\dfrac{\sqrt{3}}{4}$. He obtained the polynomial by following steps which are as shown below:

Let $\alpha = \frac{-2}{\sqrt{3}}$ and $\beta = \frac{\sqrt{3}}{4}$

Then, $\alpha + \beta = \frac{-2}{\sqrt{3}} + \frac{\sqrt{3}}{4} = \frac{-8+1}{4\sqrt{3}} = \frac{-7}{4\sqrt{3}}$

and $\alpha\beta = \frac{-2}{\sqrt{3}} \times \frac{\sqrt{3}}{4} = \frac{-1}{2}$

$\therefore$ Required polynomial $= x^2 - (\alpha + \beta)x + \alpha\beta$

$$= x^2 - \left(\frac{-7}{4\sqrt{3}}\right)x + \left(\frac{-1}{2}\right)$$

$$= x^2 + \frac{7x}{4\sqrt{3}} - \frac{1}{2}$$

$$= 4\sqrt{3}x^2 + 7x - 2\sqrt{3}$$

His another friend Kavita pointed out that the polynomial obtained is not correct.

Based on the above information, solve the following questions:

Q 1. Is the claim of Kavita correct?

Q 2. If given polynomial is incorrect, then find the correct quadratic polynomial.

Q 3. Find the value of $\alpha^2 + \beta^2$.

Q 4. What is the value of the correct polynomial, if $x = -1$?

Q 5. If correct polynomial $p(x)$ is a factor of $(x - 2)$, then find $f(2)$.

Solutions

1. Given, $\alpha = -\frac{2}{\sqrt{3}}$ and $\beta = \frac{\sqrt{3}}{4}$

$\therefore \quad \alpha + \beta = \frac{-2}{\sqrt{3}} + \frac{\sqrt{3}}{4} = \frac{-8+3}{4\sqrt{3}} = \frac{-5}{4\sqrt{3}}$

and $\alpha\beta = \frac{-2}{\sqrt{3}} \times \frac{\sqrt{3}}{4} = -\frac{1}{2}$

Yes, because value of $(\alpha + \beta)$ calculated by Anirudh is incorrect.

2. Required polynomial $= k(x^2 - (\alpha + \beta)x + \alpha\beta)$

$$= k\left(x^2 + \frac{5x}{4\sqrt{3}} - \frac{1}{2}\right)$$

$$= \frac{k}{4\sqrt{3}}(4\sqrt{3}x^2 + 5x - 2\sqrt{3})$$

$$= (4\sqrt{3}x^2 + 5x - 2\sqrt{3})$$

where $k = 4\sqrt{3}$

3. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$= \left(\frac{-5}{4\sqrt{3}}\right)^2 - 2 \times \left(\frac{-1}{2}\right) = \frac{25}{48} + 1 = \frac{73}{48}$$

Alternate method:

$$\alpha^2 + \beta^2 = \left(\frac{-2}{\sqrt{3}}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2 = \frac{4}{3} + \frac{3}{16} = \frac{64+9}{48} = \frac{73}{48}$$

4. Let correct polynomial be

$p(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

If $x = -1$, then

$$p(-1) = 4\sqrt{3}(-1)^2 + 5(-1) - 2\sqrt{3}$$

$$= 4\sqrt{3} - 5 - 2\sqrt{3} = 2\sqrt{3} - 5$$

5. We have, $p(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

Since, $p(x)$ is a factor of $(x - 2)$, then

$$p(2) = 4\sqrt{3}(2)^2 + 5(2) - 2\sqrt{3}$$

$$= 16\sqrt{3} + 10 - 2\sqrt{3} = 14\sqrt{3} + 10$$

Hence, remainder is $14\sqrt{3} + 10$.

Case Study 5

A group of school friends went on an expedition to see caves. One person remarked that the entrance of the caves resembles a parabola and can be represented by a quadratic polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, where a, b and c are real numbers.

Based on the above information, solve the following questions:

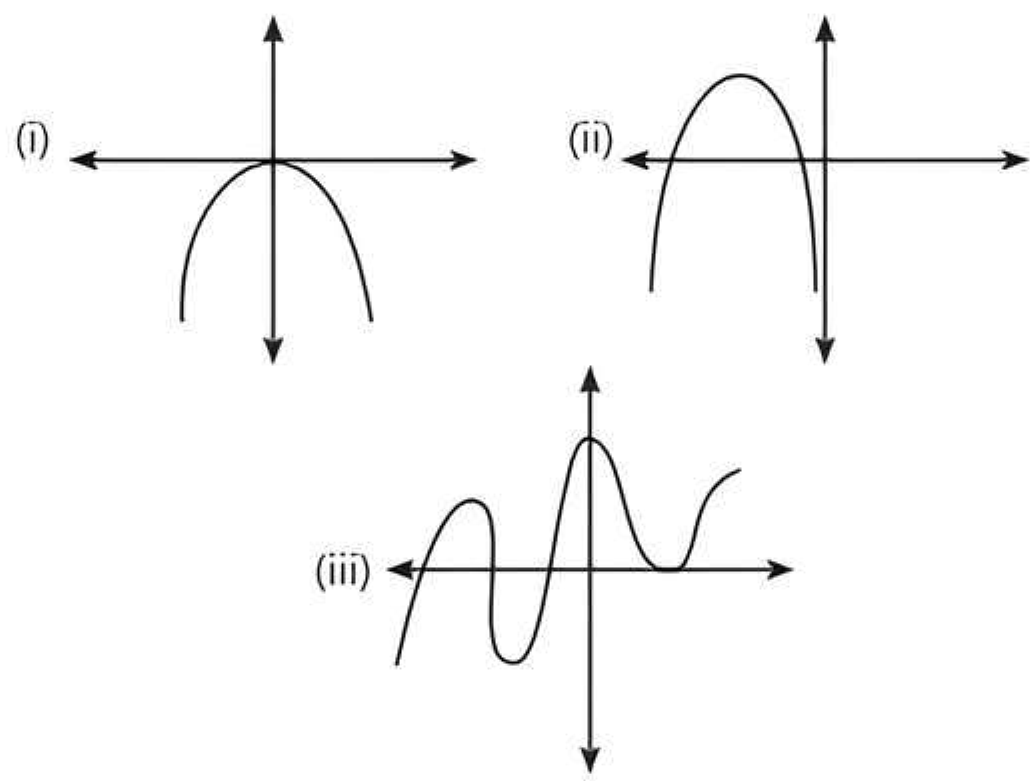
Q 1. Draw a neat labelled figure to show above situation diagrammatically.

Q 2. If one of the zeroes of the quadratic polynomial $(p - 1)x^2 + px + 1$ is 4. Find the value of p.

Q 3. Find the quadratic polynomial whose zeroes are 5 and -12.

Q 4. If one zero of the polynomial $f(x) = 5x^2 + 13x + m$ is reciprocal of the other, then find the value of m.

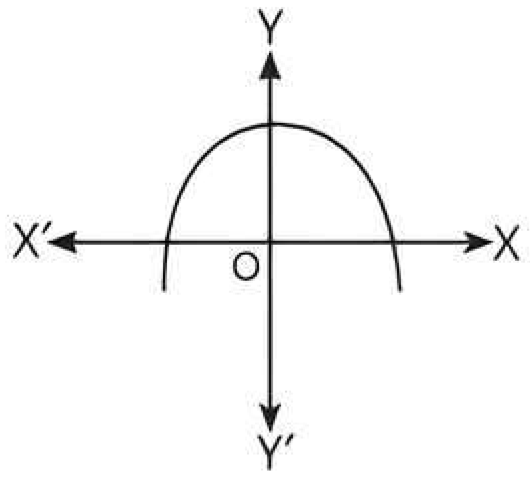
Q 5. Identify which of the following cannot be the graph of a quadratic polynomial?



Solutions

1. We have, $f(x) = ax^2 + bx + c$, $a < 0$

   It means a figure is a shape of parabola which open downwards.



2. Since, $x = 4$ is one of the zero of the polynomial $(p-1)x^2 + px + 1$.

$$\therefore \quad (p-1)(4)^2 + p(4) + 1 = 0$$
$$\Rightarrow \quad 16p - 16 + 4p + 1 = 0$$
$$\Rightarrow \quad 20p = 15$$
$$\Rightarrow \quad p = \frac{3}{4}$$

3. Required quadratic polynomial

$$= k\,[x^2 - (\text{Sum of zeroes})x + (\text{Product of zeroes})]$$
$$= k\,[x^2 - (-12 + 5)x + (-12)(5)]$$
$$= k\,[x^2 + 7x - 60], \text{ where } k \text{ is any arbitrary constant.}$$

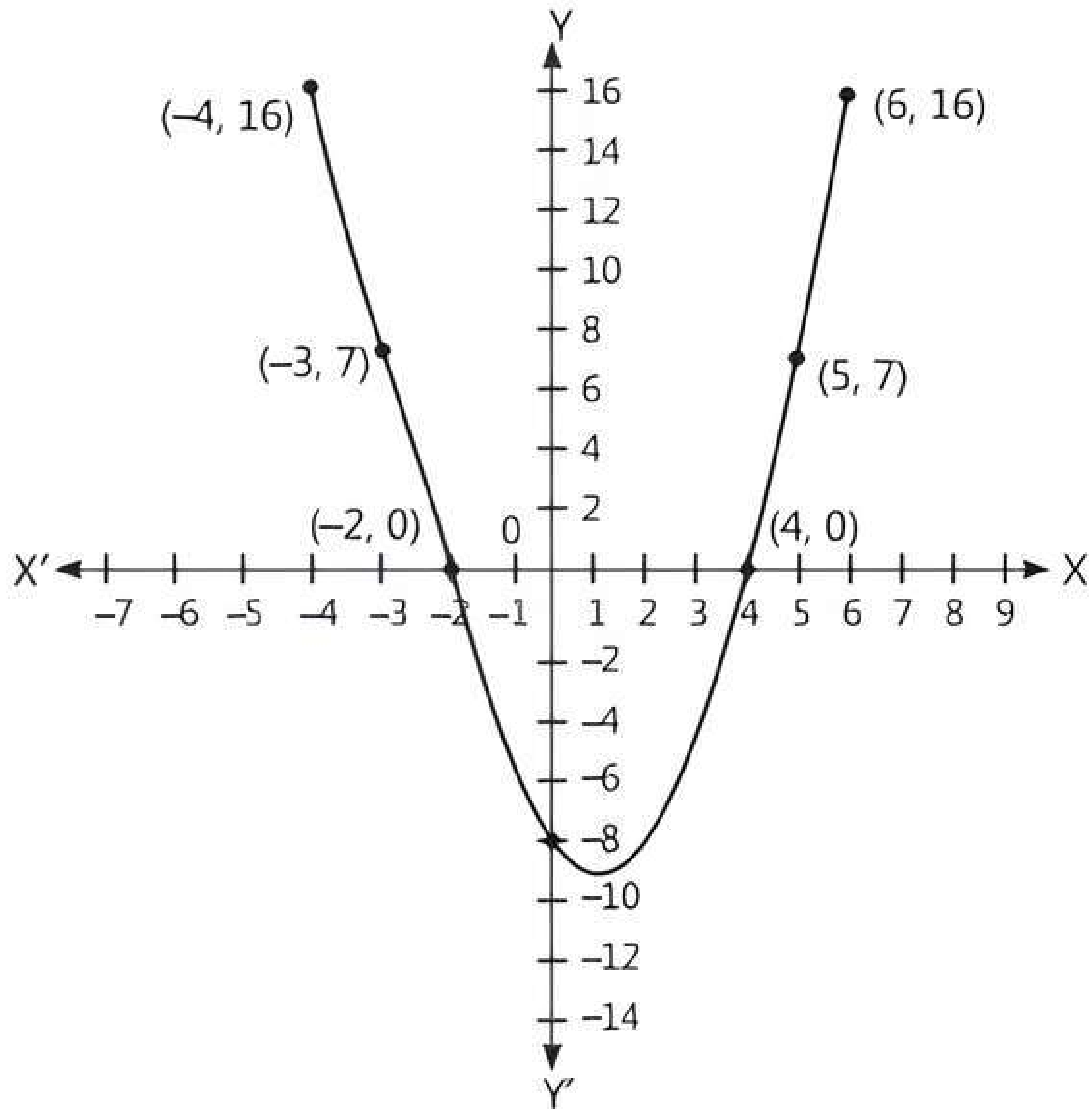
4. Since, one zero of the polynomial $f(x)$ is reciprocal of the other.

$$\therefore \text{ Product of zeroes } = (-1)^2 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$
$$\Rightarrow \quad \alpha \cdot \frac{1}{\alpha} = 1 \cdot \frac{m}{5} \quad \Rightarrow m = 5$$

5. The graph of a quadratic polynomial is always a parabola and intersects the X-axis atmost two points. Hence, graph (iii) is not a quadratic polynomial.

Case Study 6

A student was given a task to prepare a graph of quadratic polynomial $p(x) = -8 - 2x + x^2$. To draw this graph, he take seven values of y corresponding to different values of x. After plotting the points on the graph paper with suitable values, he obtain the graph as shown below.



Based on the above graph, solve the following questions:

Q 1. What is the shape of graph of a quadratic polynomial?

Q 2. Find the zeroes of given quadratic polynomial.

Q 3. From the graph, find the value of y corresponding to $x = -1$.

Q 4. The graph of the given quadratic polynomial cut at which points on the X-axis?

Q 5. The graph of the given quadratic polynomial cut at which point on Y-axis?

Solutions

1. The graph of a quadratic polynomial is a parabola which open upwards.

2. The zeroes of the quadratic polynomial $p(x) = -8 - 2x + x^2$ are x-coordinates of the points where the graph intersects the X-axis.

   From the given graph, –2 and 4 are the x-coordinates of the points where the graph of $p(x) = -8 - 2x + x^2$ intersects the X-axis.

   Hence, –2 and 4 are zeroes of $p(x) = -8 - 2x + x^2$.

3. For $x = -1$,

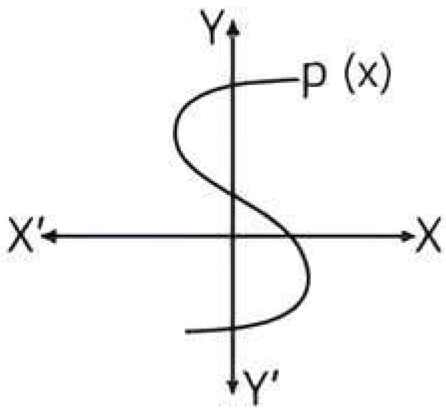
$$y = p(-1) = -8 - 2(-1) + (-1)^2$$
$$= -8 + 2 + 1$$
$$= -8 + 3 = -5$$

4. The graph of the given quadratic polynomial cut X-axis at points (–2, 0) and (4, 0).

5. The graph of the given quadratic polynomial cut Y-axis at point (0, –8).

Very Short Answer Type Questions

Q 1. In figure, the graph of a polynomial $p(x)$ is shown. Find the number of zeroes of $p(x)$. *[CBSE 2016]*



Q 2. If the sum of the zeroes of the polynomial $p(x) = (k^2 - 14)\,x^2 - 2x - 12$ is 1, then find the value of k. *[CBSE 2017]*

Q 3. Find the polynomial whose zeroes are $\sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}$.

Q 4. If α and β are the zeroes of a polynomial such that $\alpha + \beta = -6$ and $\alpha\beta = 5$, then find the polynomial. *[CBSE 2016]*

Q 5. If both the zeroes of the quadratic polynomial $ax^2 + bx + c$ are equal and opposite in sign, then find the value of b. *[V. Imp.]*

Q 6. If one of the zeroes of the quadratic polynomial $(k - 1)\,x^2 + kx + 1$ is -3, then find the value of k. *[V. Imp.]*

Q 7. If $(x - a)$ is a factor of $p(x) = x^3 - ax^2 + 6 - a$, then find $p(a)$.

Q 8. If one zero of the polynomial $p(x) = 6x^2 + 37x - (k - 2)$ is reciprocal of the other, then find the value of k. *[CBSE 2023]*

Short Answer Type-I Questions

Q 1. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answers given by the students: *[CBSE 2020]*

$2x + 3,\ 3x^2 + 7x + 2,\ 4x^3 + 3x^2 + 2,\ x^3 + \sqrt{3x} + 7,$

$7x + \sqrt{7},\ 5x^3 - 7x + 2,\ 2x^2 + 3 - \frac{5}{x},\ 5x - \frac{1}{2},$

$ax^3 + bx^2 + cx + d,\ x + \frac{1}{x}$

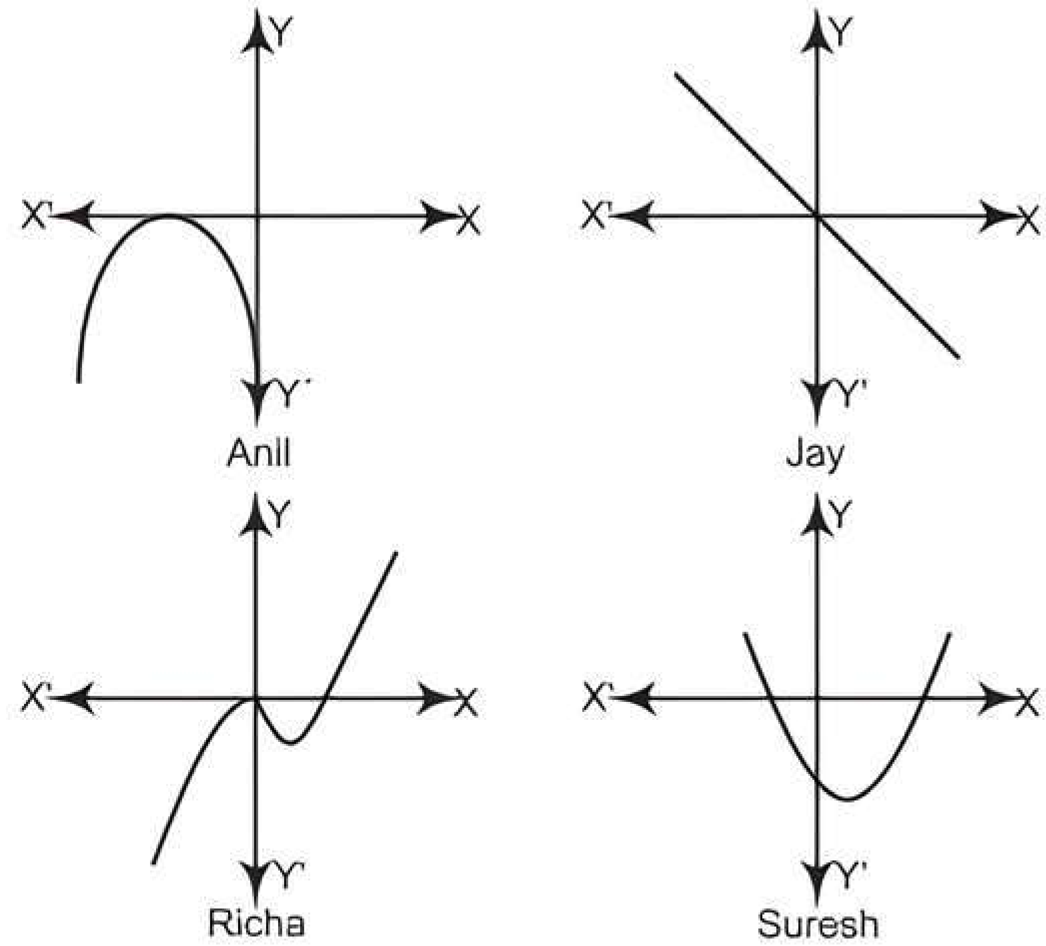
Answer the following questions:

(i) How many of the above ten, are not polynomials?

(ii) How many of the above ten, are quadratic polynomials?

Q 2. Read the following passage and answer the questions that follows:

In a classroom, four students Anil, Jay, Richa and Suresh were asked to draw the graph of $y = ax^2 + bx + c$. Following graphs are drawn by the students:



(i) How many students have drawn the graph correctly?

(ii) Which type of polynomial is represented by Jay's graph?

Q 3. Find a quadratic polynomial whose zeroes are $\frac{3+\sqrt{5}}{5}$ and $\frac{3-\sqrt{5}}{5}$.

Q 4. Find the condition that zeroes of polynomial $p(x) = ax^2 + bx + c$ are reciprocal of each other. *[CBSE 2017]*

Q 5. If α and β are the zeroes of $p(x) = 4x^2 + 3x + 7$, then find $\frac{1}{\alpha} + \frac{1}{\beta}$.

Q 6. If α and β are the zeroes of the polynomial $x^2 - 5x + k$ such that $\alpha - \beta = 1$, find the value of k.

Q 7. If α, β are zeroes of quadratic polynomial $5x^2 + 5x + 1$, find the value of:

(i) $\alpha^2 + \beta^2$ (ii) $\alpha^{-1} + \beta^{-1}$ *[CBSE SQP 2023-24]*

Q 8. If $(x + a)$ is a factor of $2x^2 + 2ax + 5x + 10$, then find a. *[CBSE 2016]*

Q 9. If one zero of the polynomial $x^2 + 11x + k$ is -3, then find the value of k and the other zero. *[CBSE 2016]*

Short Answer Type-II Questions

Q 1. Find the zeroes of the given quadratic polynomial $f(x) = 2x^2 - x - 6$ and verify the relationship between the zeroes and the coefficients. *[CBSE 2017]*

Q 2. Find the quadratic polynomial, sum and product of whose zeroes are -1 and -20 respectively. Also, find the zeroes of the polynomial so obtained. *[CBSE 2019]*

Q 3. If one zero of polynomial $(a^2 + 9)\,x^2 + 13x + 6a$ is reciprocal of the other, find the value of a. *[CBSE 2015]*

Q 4. If the zeroes of the polynomial $x^2 + px + q$ are double in the value to the zeroes of the polynomial $2x^2 - 5x - 3$, then find the values of p and q. *[CBSE SQP 2022-23; CBSE 2016]*

Q 5. If the sum of the zeroes of the polynomial $p(x) = (a + 1)x^2 + (2a + 3)x + (3a + 4)$ is -1, then find the product of its zeroes.

Q 6. If α and β are the zeroes of $x^2 - x - 2$, form a quadratic polynomial whose zeroes are $2\alpha + 1, 2\beta + 1$. *[CBSE 2016]*

Q 7. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 4x + 3$, find the value of $(\alpha^4\beta^2 + \alpha^2\beta^4)$. *[CBSE 2019]*

Q 8. Verify whether 2,3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x) = 2x^3 - 11x^2 + 17x - 6$. *[CBSE 2017]*

Long Answer Type Questions

Q 1. Find all zeroes of the polynomial $3x^3 + 10x^2 - 9x - 4$, if one of its zeroes is 1. *[CBSE 2019]*

Q 2. Find all the zeroes of the polynomial $x^4 + x^3 - 14x^2 - 2x + 24$, if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$. *[CBSE 2019]*

Q 3. Find the zeroes of the quadratic polynomial $f(x) = abx^2 + (b^2 - ac)x - bc$, and verify the relationship between the zeroes and its coefficients.

Q 4. If α and β are the zeroes of the quadratic polynomial $f(x) = kx^2 + 4x + 4$ such that $\alpha^2 + \beta^2 = 24$, find the value of k.

Solutions

Very Short Answer Type Questions

1.

TR!CK

The zeroes of the polynomial are the x-coordinates of those points where its graph touches or intersects the X-axis.

The number of zeroes of $p(x)$ is 1 as the graph intersects X-axis at one point.

2. Given, $p(x) = (k^2 - 14)x^2 - 2x - 12$

Also, sum of zeroes = 1 (given)

$\Rightarrow \quad -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = 1$

$\Rightarrow \quad \frac{-(-2)}{k^2 - 14} = 1$

$\Rightarrow \quad k^2 - 14 = 2$

$\Rightarrow \quad k^2 = 16$

$\Rightarrow \quad k = \pm 4$

3. The given zeroes are $\sqrt{\frac{3}{2}}$ $\sqrt{\frac{3}{2}}$

Required quadratic polynomial.

$p(x) = k[x^2 - (\text{Sum of zeroes})\, x + \text{Product of zeroes}]$

where k is the arbitrary constant.

$$= k\left[x^2 - \left(\sqrt{\frac{3}{2}} - \sqrt{\frac{3}{2}}\right)x + \left(\sqrt{\frac{3}{2}}\right)\cdot\left(-\sqrt{\frac{3}{2}}\right)\right]$$

$$= k\left[x^2 - 0\cdot x - \frac{3}{2}\right] = k\left(x^2 - \frac{3}{2}\right)$$

$\therefore \quad p(x) = \frac{2}{2}(2x^2 - 3) = 2x^2 - 3$, where $k = 2$.

4. Given, $\alpha + \beta = -6$ and $\alpha\beta = 5$.

Required quadratic polynomial.

$p(x) = k[x^2 - (\text{sum of zeroes})\, x + \text{product of zeroes}]$

where k is the arbitrary constant.

$= k[x^2 - (\alpha + \beta)\, x + \alpha\beta] = k[x^2 - (-6)\, x + 5]$

$\therefore \; p(x) = x^2 + 6x + 5$, where $k = 1$.

5. Let α and $-\alpha$ be the zeroes of given polynomial.

$\because \quad$ Sum of zeroes $= -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$\therefore \quad \alpha + (-\alpha) = \frac{-b}{a}$

$\Rightarrow \quad 0 = \frac{-b}{a} \Rightarrow b = 0$

6. Given, one zero of polynomial is -3.

TiP

A real number a is said to be a zero of a polynomial $p(x) = ax^2 + bx + c$, if $a\alpha^2 + b\alpha + c = 0$.

On substituting -3 in given polynomial, we get

$(k - 1)(-3)^2 + k(-3) + 1 = 0$

$\Rightarrow \quad 9k - 9 - 3k + 1 = 0$

$\Rightarrow \quad 6k - 8 = 0 \quad \Rightarrow \quad 6k = 8$

$\Rightarrow \quad k = \frac{8}{6} \quad \Rightarrow \quad k = \frac{4}{3}$

7. Given, $p(x) = x^3 - ax^2 + 6 - a$

Since, $(x - a)$ is a factor of $p(x)$.

Therefore, $p(a) = a^3 - a \times a^2 + 6 - a$

$= a^3 - a^3 + 6 - a = 6 - a$

8. Given polynomial, $p(x) = 6x^2 + 37x - (k - 2)$

Let α and β be the zeroes of $p(x)$.

According to question, $\beta = \frac{1}{\alpha}$

$\therefore$ Product of zeroes $= (-1)^2 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\Rightarrow \quad \alpha \cdot \frac{1}{\alpha} = 1 \times \frac{-(k-2)}{6}$

$\Rightarrow \quad 6 = -k + 2 \Rightarrow k = 2 - 6 = -4$

Short Answer Type-I Questions

1. (i) $x^3 + \sqrt{3x} + 7,\ 2x^2 + 3 - \frac{5}{x}$ and $x + \frac{1}{x}$ are not polynomials, because these polynomials contains negative integral powers or fractional powers. So, there are 3 expressions which are not polynomials.

(ii)

TiP

A polynomial whose degree is 2, is a quadratic polynomial. General form of quadratic polynomial is $ax^2 + bx + c, a \neq 0$.

$3x^2 + 7x + 2$ is only a quadratic polynomial. So, there are only 1 quadratic polynomial.

2. (i)

TR!CK

For any quadratic polynomial $ax^2 + bx + c$, the graph of the corresponding equation has one of the two shapes either open upwards like ∪ or open downwards like ∩.

So, Anil and Suresh have drawn the correct graph. Hence, two students have drawn the graph correctly.

(ii) Linear polynomial is represented by Jay's graph.

3. ∵ Sum of zeroes $= \frac{3+\sqrt{5}}{5} + \frac{3-\sqrt{5}}{5}$

$$= \frac{3+\sqrt{5}+3-\sqrt{5}}{5} = \frac{6}{5}$$

and product of zeroes $= \left(\frac{3+\sqrt{5}}{5}\right)\left(\frac{3-\sqrt{5}}{5}\right)$

$$= \frac{(3)^2-(\sqrt{5})^2}{25} = \frac{9-5}{25} = \frac{4}{25}$$

∴ Required quadratic polynomial,

$p(x) = k[x^2 - (\text{sum of zeroes})\, x + \text{product of zeroes}]$

$$= k\left[x^2 - \left(\frac{6}{5}\right)x + \left(\frac{4}{25}\right)\right] = \frac{k}{25}(25x^2 - 30x + 4)$$

$\Rightarrow p(x) = (25x^2 - 30x + 4)$

where, $k = 25 \in R$ is an arbitrary constant.

4. Let one zero of the polynomial $p(x)$ be α. Then other will be $\frac{1}{\alpha}$.

Now, product of zeroes $= (-1)^2 \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$\Rightarrow \quad \alpha \cdot \frac{1}{\alpha} = 1 \cdot \frac{c}{a} \Rightarrow 1 = \frac{c}{a}$$

$$\therefore \quad a = c$$

Hence, required condition is

Coefficient of x^2 = Constant term

5. Given, $p(x) = 4x^2 + 3x + 7$

Sum of zeroes $= \alpha + \beta = (-1).\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{3}{4}$

Product of zeroes $= \alpha\beta = (-1)^2 \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{7}{4}$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{-3/4}{7/4} = -\frac{3}{7}$

6. Given, $p(x) = x^2 - 5x + k$ and its zeroes are α and β.

Sum of zeroes $= \alpha + \beta = (-1)\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$\therefore \quad \alpha + \beta = \frac{-(-5)}{1} = 5 \quad \text{...(1)}$$

Product of zeroes $= \alpha\beta = (-1)^2.\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$\therefore \quad \alpha\beta = \frac{k}{1} = k \quad \text{...(2)}$$

But $\quad \alpha - \beta = 1 \quad$ (given) ...(3)

Using identity,

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$\Rightarrow \quad (1)^2 = (5)^2 - 4k$$

[from eqs. (1), (2) and (3)]

$$\Rightarrow \quad 1 = 25 - 4k$$

$$\Rightarrow \quad 4k = 24$$

$$\therefore \quad k = \frac{24}{4} = 6$$

7. Given, α and β are the zeroes of the polynomial

$$p(x) = 5x^2 + 5x + 1.$$

$$\therefore \quad \alpha + \beta = (-1).\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{5}{5} = -1$$

and $\quad \alpha\beta = (-1)^2.\frac{\text{Constant term}}{\text{Coefficient of } x^2} = 1 \times \frac{1}{5} = \frac{1}{5}$

(i) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\,\alpha\beta = (-1)^2 - 2\left(\frac{1}{5}\right) = 1 - \frac{2}{5} = \frac{3}{5}$

(ii) $\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-1}{(1/5)} = -5$

8. Given, $(x + a)$ is a factor of $2x^2 + 2ax + 5x + 10$.

Then, $x + a = 0$ must satisfy $2x^2 + 2ax + 5x + 10 = 0$.

Consider, $\quad x + a = 0$

i.e., $\quad x = -a$

On substituting the value of 'x' in given polynomial, we get

$$2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$$

$$\Rightarrow \quad 2a^2 - 2a^2 - 5a + 10 = 0$$

$$\Rightarrow \quad -5a = -10 \Rightarrow a = \frac{-10}{-5}$$

$$\Rightarrow \quad a = 2$$

9. One zero of given polynomial is -3.

TiP

A real number α is said to be a zero of a polynomial $p(x)$, if $p(\alpha) = 0$.

On substituting -3 in given polynomial, we get

$$(-3)^2 + 11(-3) + k = 0$$

$\Rightarrow \quad 9 - 33 + k = 0$

$\Rightarrow \quad k - 24 = 0$

$\Rightarrow \quad k = 24$

$\therefore$ Polynomial $p(x) = x^2 + 11x + 24$

$= x^2 + 8x + 3x + 24$

(by splitting the middle term)

$= x(x + 8) + 3(x + 8)$

$= (x + 8)(x + 3)$

For zeroes, $(x + 3)(x + 8) = 0$

$\Rightarrow \quad x + 3 = 0 \quad$ or $\quad x + 8 = 0$

$\Rightarrow \quad x = -3 \quad$ or $\quad x = -8$

Hence, the other zero is -8.

Short Answer Type-II Questions

1. Given, $f(x) = 2x^2 - x - 6$

$= 2x^2 - 4x + 3x - 6$

$= 2x(x - 2) + 3(x - 2)$

$= (x - 2)(2x + 3)$

For zeroes, $f(x) = 0$

$\therefore \quad (x - 2)(2x + 3) = 0$

$\Rightarrow \quad x = 2,\ x = \frac{-3}{2}$

Verification:

Sum of zeroes $= 2 + \left(\frac{-3}{2}\right) = \frac{4-3}{2} = \frac{1}{2} = \frac{-(-1)}{2}$

$= (-1) \cdot \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of zeroes $= 2 \times \left(\frac{-3}{2}\right) = \frac{(-6)}{2}$

$= (-1)^2 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence, the relationship between the zeroes and the coefficients is verified.

2. If α and β are the zeroes of the quadratic polynomial $f(x)$, then

$f(x) = x^2 - (\alpha + \beta)x + \alpha \cdot \beta \quad ...(1)$

Given, sum of zeroes $= \alpha + \beta = -1$

and product of zeroes $= \alpha\beta = -20$

$\therefore$ From eq. (1),

$f(x) = x^2 - (-1)x - 20 = x^2 + x - 20$

$= x^2 + 5x - 4x - 20$

$= x(x + 5) - 4(x + 5)$

$= (x + 5)(x - 4)$

The value of $f(x)$ will be zero, when $x + 5 = 0$ or $x - 4 = 0$

i.e., when $x = -5$ or $x = 4$

So, the zeroes of $f(x)$ are -5 and 4.

3. Given, $p(x) = (a^2 + 9)x^2 + 13x + 6a$.

Let one zero be α, then other zero will be $\frac{1}{\alpha}$.

$\therefore$ Product of zeroes $= \alpha \times \frac{1}{\alpha} = (-1)^2 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\Rightarrow \quad 1 = \frac{6a}{a^2 + 9}$

or $\quad a^2 + 9 = 6a$

$\Rightarrow \quad a^2 - 6a + 9 = 0 \Rightarrow a^2 - 2 \times 3 \times a + (3)^2 = 0$

TR!CK

$x^2 - 2xy + y^2 = (x - y)^2$

$\Rightarrow \quad (a - 3)^2 = 0$

$\Rightarrow \quad a = 3$

Hence, the value of a is 3.

4. Given, $p(x) = 2x^2 - 5x - 3 = 2x^2 - 6x + x - 3$

TR!CK

Product, $2 \times 3 = 6$

$\because \quad 6 = 6 \times 1 = 3 \times 2$

$\therefore$ *We take 6 and 1 as a factors of 6.*

So, middle term $-5 = -6 + 1$.

$\Rightarrow \quad p(x) = 2x(x - 3) + 1(x - 3) = (x - 3)(2x + 1)$

For zeroes, $(x - 3)(2x + 1) = 0$

$x - 3 = 0 \quad$ or $\quad 2x + 1 = 0$

$\Rightarrow \quad x = 3 \quad$ or $\quad x = -\frac{1}{2}$

Now, zeroes of the required polynomial are

$3 \times 2,\ \left(-\frac{1}{2} \times 2\right)$ *i.e.*, $6, -1$

($\because$ zeroes of $x^2 + px + q$ are double.)

$\therefore$ Sum of zeroes $= 6 + (-1) = 5$

and product of zeroes $= 6 \times (-1) = -6$.

So, required quadratic polynomial

$= x^2 - (\text{Sum of zeroes})\,x + \text{Product of zeroes}$

$= x^2 - 5x - 6$

On comparing with $x^2 + px + q$, we get

$p = -5$ and $q = -6$.

5. Given, $p(x) = (a + 1)x^2 + (2a + 3)x + (3a + 4)$

Let α and β be the zeroes of the given polynomial $p(x)$.

$\therefore$ Sum of zeroes $= \alpha + \beta$

$= (-1) \cdot \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -1 \quad$ (given)

$\Rightarrow \quad -\frac{(2a + 3)}{(a + 1)} = -1$

$\Rightarrow \quad 2a + 3 = a + 1$

$\Rightarrow \quad 2a - a = 1 - 3$

$\Rightarrow \quad a = -2$

On substituting the value of 'a' in $p(x)$, we get

$p(x) = (-2 + 1)x^2 + (2 \times -2 + 3)x + (3 \times -2 + 4)$

$= -x^2 + (-4 + 3)x + (-6 + 4)$

$= -x^2 - x - 2$

So, product of its zeroes

$= (-1)^2 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-2}{-1} = 2$

6. Let polynomial $f(x) = x^2 - x - 2$

As, α and β are the zeroes of $f(x)$.

$\therefore$ Sum of zeroes $= \alpha + \beta$

$= (-1) \cdot \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{(-1)}{1} = 1$

and product of zeroes $= \alpha\beta$

$= (-1)^2 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{(-2)}{1} = -2$

For the given zeroes $(2\alpha + 1)$ and $(2\beta + 1)$,

Sum of zeroes $= (2\alpha + 1) + (2\beta + 1) = 2(\alpha + \beta) + 2$

$= 2(1) + 2 = 2 + 2 = 4$

and product of zeroes $= (2\alpha + 1) \cdot (2\beta + 1)$

$= 4\alpha\beta + 2\beta + 2\alpha + 1$

$= 4\alpha\beta + 2(\alpha + \beta) + 1$

$= 4(-2) + 2(1) + 1$

$= -8 + 2 + 1 = -5$

Hence, the required polynomial,

$p(x) = k[x^2 - (\text{Sum of zeroes})\, x + (\text{Product of zeroes})]$

where, k is an arbitrary constant.

$p(x) = k[x^2 - \{(2\alpha + 1) + (2\beta + 1)\}x - (2\alpha + 1) \cdot (2\beta + 1)\}]$

$= k(x^2 - 4x - 5)$

7. Given, $f(x) = x^2 - 4x + 3$

On comparing with $ax^2 + bx + c$, we get

$a = 1, b = -4, c = 3$

Sum of zeroes $= \alpha + \beta = (-1) \cdot \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$= \frac{-b}{a} = \frac{-(-4)}{1} = 4$

and product of zeroes $= \alpha\beta = (-1)^2 \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$= \frac{c}{a} = \frac{3}{1} = 3$

Now, $\alpha^4\beta^2 + \alpha^2\beta^4 = \alpha^2\beta^2(\alpha^2 + \beta^2)$

$= \alpha^2\beta^2[(\alpha + \beta)^2 - 2\alpha\beta]$

TR!CK

$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$= (\alpha\beta)^2 [(\alpha + \beta)^2 - 2\alpha\beta]$

$= (3)^2 [(4)^2 - 2(3)] = 9[16 - 6]$

$= 9(10) = 90$

8. Given, $p(x) = 2x^3 - 11x^2 + 17x - 6$

TiP

A real number k is zero of a polynomial f(x), if f(k) = 0.

Now, $p(2) = 2(2)^3 - 11(2)^2 + 17(2) - 6$

$= 2 \times 8 - 11 \times 4 + 17 \times 2 - 6$

$= 16 - 44 + 34 - 6 = 50 - 50 = 0$

$p(3) = 2(3)^3 - 11(3)^2 + 17(3) - 6$

$= 2 \times 27 - 11 \times 9 + 17 \times 3 - 6$

$= 54 - 99 + 51 - 6 = 105 - 105 = 0$

and $p\left(\frac{1}{2}\right)^3 = 2\left(\frac{1}{2}\right)^3 - 11\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) - 6$

$= 2 \times \frac{1}{8} - 11 \times \frac{1}{4} + \frac{17}{2} - 6$

$= \frac{1}{4} - \frac{11}{4} + \frac{17}{2} - 6 = \frac{1 - 11 + 34 - 24}{4}$

$= \frac{35 - 35}{4} = \frac{0}{4} = 0$

Hence, 2, 3 and $\frac{1}{2}$ are the zeroes of $p(x)$.

Long Answer Type Questions

1. Let $p(x) = 3x^3 + 10x^2 - 9x - 4$

As, it is given that 1 is a zero of $p(x)$.

Therefore, $(x - 1)$ is a factor of $p(x)$.

$\therefore \quad p(x) = 3x^2(x - 1) + 13x(x - 1) + 4(x - 1)$

$= (x - 1)(3x^2 + 13x + 4)$

Now, $3x^2 + 13x + 4 = 3x^2 + (12 + 1)x + 4$

TR!CK

Product, $3 \times 4 = 12$

$\because \quad 12 = 3 \times 4 = 2 \times 6 = 12 \times 1$

$\therefore$ *We take 12 and 1 as a factors of 12.*

So, middle term $13 = 1 + 12$

$= (3x^2 + 12x) + (x + 4)$

$= 3x(x + 4) + 1(x + 4) = (3x + 1)(x + 4)$

To find the other zeroes of $p(x)$,

Put $(3x + 1)(x + 4) = 0 \Rightarrow x = -\frac{1}{3}, -4$

Hence, all zeroes of $p(x)$ are 1, −1/3 and −4.

2. Let $p(x) = x^4 + x^3 - 14x^2 - 2x + 24$.

Two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$

Let $\sqrt{2} = \alpha$ and $-\sqrt{2} = \beta$

Now, $\alpha + \beta = \sqrt{2} + (-\sqrt{2}) = 0$

and $\alpha\beta = \sqrt{2} \times (-\sqrt{2}) = -2$

So, quadratic polynomial is

$q(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (0)x + (-2)$

$= x^2 - 2$

$p(x) = x^2(x^2 - 2) + x(x^2 - 2) - 12(x^2 - 2)$

$= (x^2 - 2)(x^2 + x - 12)$

Now, $x^2 + x - 12 = x^2 + 4x - 3x - 12$

$= x(x + 4) - 3(x + 4)$

$= (x - 3)(x + 4)$

To find the zeroes, put $(x - 3)(x + 4) = 0$

$\Rightarrow \quad x = 3, -4$

Hence, all zeroes of $p(x)$ are $-\sqrt{2}, \sqrt{2}, -4$ and 3.

3. Given, $f(x) = abx^2 + (b^2 - ac)x - bc$

$\Rightarrow \quad f(x) = abx^2 + b^2x - acx - bc$

$\Rightarrow \quad f(x) = bx(ax+b) - c(ax+b)$

$\Rightarrow \quad f(x) = (ax+b)(bx-c)$

The zeroes of $f(x)$ are given by $f(x) = 0$.

Now, $\quad f(x) = 0$

$\Rightarrow \quad (ax+b)\ (bx-c) = 0$

$\Rightarrow \quad ax+b = 0$ or $bx - c = 0$

$\Rightarrow \quad x = -\frac{b}{a}$ or $x = \frac{c}{b}$

Thus, the zeroes of $f(x)$ are $\alpha = -\frac{b}{a}$ and $\beta = \frac{c}{b}$.

Verification:

Now, $\alpha + \beta = -\frac{b}{a} + \frac{c}{b} = \frac{ac - b^2}{ab}$ and $\alpha\beta = -\frac{b}{a} \times \frac{c}{b} = -\frac{c}{a}$

Also, $(-1)\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\left(\frac{b^2 - ac}{ab}\right) = \frac{ac - b^2}{ab}$

and $\quad (-1)^2 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^2} = -\frac{bc}{ab} = -\frac{c}{a}$

Hence, sum of the zeroes $= \alpha + \beta$

$$= (-1) \cdot \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

and product of the zeroes $= \alpha . \beta$

$$= (-1)^2 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, it is verified.

4. Given that α and β are the zeroes of the quadratic polynomial $f(x) = kx^2 + 4x + 4$.

$\therefore \quad \alpha + \beta = -\frac{4}{k}$ and $\alpha\beta = \frac{4}{k}$

Also, given $\quad \alpha^2 + \beta^2 = 24$

TR!CK

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$\therefore \quad (\alpha+\beta)^2 - 2\alpha\beta = 24$

$\Rightarrow \quad \left(-\frac{4}{k}\right)^2 - 2 \times \frac{4}{k} = 24$

$\Rightarrow \quad \frac{16}{k^2} - \frac{8}{k} = 24$

$\Rightarrow \quad 16 - 8k = 24k^2$

$\Rightarrow \quad 3k^2 + k - 2 = 0$

$\Rightarrow \quad 3k^2 + 3k - 2k - 2 = 0$

$\Rightarrow \quad 3k(k+1) - 2(k+1) = 0$

$\Rightarrow \quad (k+1)(3k-2) = 0$

$\Rightarrow \quad k + 1 = 0$ or $3k - 2 = 0$

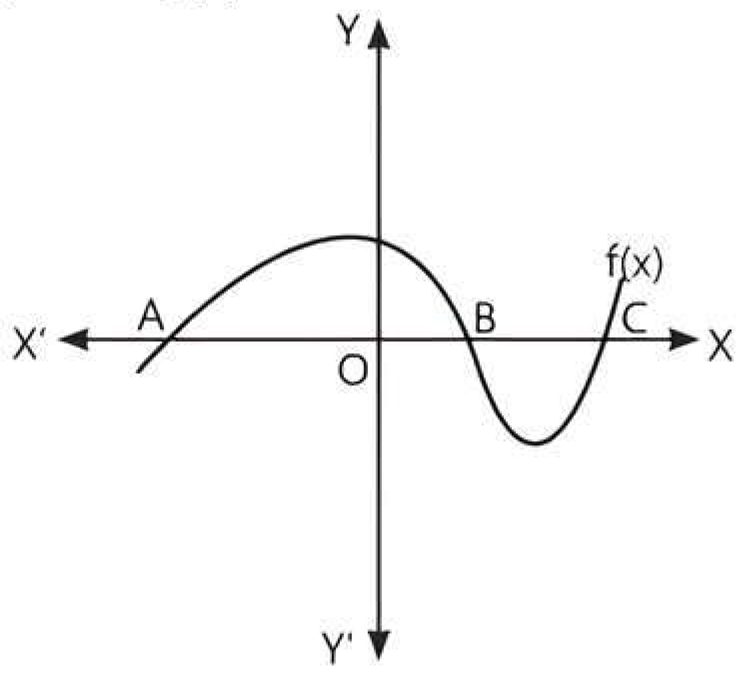
$\Rightarrow \quad k = -1$ or $k = 2/3$

Hence, $\quad k = -1$ or $k = 2/3$

Chapter Test

Multiple Choice Questions

Q 1. In the given figure, the number of zeroes of the polynomial $f(x)$ are:



a. 3 b. 2

c. 1 d. 0

Q 2. If α and β are the zeroes of the polynomial $p(x) = x^2 - 2x - 3$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to:

a. $\frac{-2}{3}$ b. $-\frac{1}{3}$

c. $\frac{1}{3}$ d. $\frac{2}{3}$

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): If one zero of polynomial $p(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of other, then $k = 3$.

Reason (R): If $(x - \alpha)$ is a factor of $p(x)$, then $p(\alpha) = 0$.

Q 4. Assertion (A): If two zeroes of the polynomial $f(x) = x^3 - 2x^2 - 3x + 6$ are $\sqrt{3}$ and $-\sqrt{3}$, then its third zero is 4.

Reason (R): If α, β and γ are zeroes of the polynomial $f(x) = ax^3 + bx^2 + cx + d$. Then,

$$\textbf{Sum of the zeroes} = (-1) \cdot \frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

Fill in the Blanks

Q 5. The number of quadratic polynomial whose zeroes are 3 and –5, is ....

Q 6. The parabola representing a quadratic polynomial $f(x) = ax^2 + bx + c$ opens upward when ....

True/False

Q 7. A cubic polynomial can have atmost three zeroes.

Q 8. The zeroes of polynomial $p(x)$ are precisely the x-coordinate of the points, where the graph of $p(x)$ intersect the X-axis.

Case Study Based Question

Q 9. Naveeka everyday goes to swimming. One day, Naveeka noticed the water coming out of the pipes to fill the pool. She carefully told her brother that the shape of the path of the water coming is like a parabola and also that a parabola can be represented by a quadratic polynomial which has atmost two zeroes.

Based on the above information, solve the following questions:

(i) The number of zeroes that polynomial $f(x) = (x + 2)^2 - 16$ can have is:

a. 0 b. 1 c. 2 d. 3

(ii) If the product of the zeroes of the quadratic polynomial $f(x) = ax^2 - 6x - 6$ is 4, then the value of a is:

a. $\frac{-3}{2}$ b. $\frac{3}{2}$ c. $\frac{2}{3}$ d. $\frac{-2}{3}$

(iii) The flow of the water in the pool is represented by $x^2 - 2x - 8$, then its zeroes are:

a. 2, –4 b. 4, –2
c. –2, –2 d. –4, –4

(iv) If α and β are the zeroes of the polynomial x^2-1, then the value of $\frac{1}{\alpha}+\frac{1}{\beta}$ is:

a. 0 b. $\frac{1}{2}$ c. 1 d. –1

(v) A quadratic polynomial whose one zero is –3 and product of zeroes is 0, is:

a. $3x^2 + 3$ b. $x^2 - 3x$
c. $x^2 + 3x$ d. $3x - 3$

Very Short Answer Type Questions

Q 10. Find the value of k, if the product of the zeroes of $x^2 - 3kx + 2k^2 - 1$ is 7.

Q 11. If the sum of the zeroes of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x + 5$ is 6, then find the value of k.

Short Answer Type-I Questions

Q 12. If the zeroes of the polynomial $x^2 + px + q$ are triple in value to the zeroes of $2x^2 - 5x - 3$, then find the values of p and q.

Q 13. If α and β are the zeroes of the polynomial $f(x) = 2x^2 + 5x + k$ satisfying the relation $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k for this to be possible.

Short Answer Type-II Questions

Q 14. It is given that 1 is one of the zero of the polynomial $7x - x^3 - 6$. Find its other zeroes.

Q 15. If α and β are the zeroes of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$, find a polynomial whose zeroes are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.

Long Answer Type Question

Q 16. Find the zeroes of the polynomial $p(x) = x^2 + 2\sqrt{2}x - 6$ and also verify the relation between the zeroes and their coefficients.